

SYLLABUS FOR MATHEMATICS (MSc)



**DEPARTMENT OF MATHEMATICS
NAGALAND UNIVERSITY
LUMAMI**

(Approved by the 40th Board of School of Sciences)

Programme Details

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|-----------------------------|-------------------------|
| 1. Name of the Degree | : MSc |
| 2. Program Code | : MAT |
| 3. Name of Specialisation | : Mathematics |
| 4. Intake capacity | : 15 |
| 5. Introduction of Syllabus | : 2013 |
| 6. Year of Revision | : 2023 |
| 7. Duration | : 2 years (4 semesters) |

Course Structure

Abbreviations: **T** = Contact hours (Theory), **P** = Contact hours (Practical), **CR** = Credits,
MATC = Mathematics Compulsory paper, **MATCBCP**= Choice Based Credit paper.

Semester	Course Code	Course Title	Total Marks	CR
I	MATC 101	Analysis on Metric Spaces	100	5
	MATC 102	Linear Algebra	100	5
	MATC 103	Ordinary Differential Equations	100	4
	MATC 104	Partial Differential Equations	50	2
	MATC 105	Analysis on $\mathbb{R}$	100	4
Total			450	20
II	MATC 201	Algebra	100	5
	MATC 202	Complex Function Theory	100	4
	MATC 203	Fourier Series	50	2
	MATC 204	Topology	100	5
	MATC 205	Several Variable Calculus	100	4
Total			450	20
III	MATC 301	Topological Groups	100	4
	MATC 302	Programming in C (Practical)	50	2
	MATC 303	Measure Theory	100	5
	MATC 304	Elementary Number Theory	100	4
	MATC 305	Theory of Field Extensions	100	
Total			450	20
IV	MATC 401	Numerical Analysis	100	4
	MATC 402	Computer Oriented Num. Analysis Practical	50	2
	MATC 403	Functional Analysis	100	5
	MATC 404	Project Work	100	4
	MATCBCP4XY	Choice Based Credit Paper (CBCP)	100	5
Total			450	20
Grand Total			1800	80

Choice Based Credit Papers

MATCBCP401: Ring Theory (5 Credits)

MATCBCP402: Algebraic Topology (5 Credits)

MATCBCP403: Mathematical Methods (5 Credits)

MATCBCP404: Commutative Algebra (5 Credits)

MATCBCP405: Stone-Cech Compactification of a Discrete Semigroup (5 Credits)

Note:

1. The choice based credit papers will be offered subject to the availability of faculty specialised in the concerned field.
2. MATCBCP4XY will correspond to the Course Code of the choice-based credit papers being offered in the fourth semester.
3. Students may take desired papers from SWAYAM MOOCS for the choice based credit papers and may transfer the credits at the end of the course.

Programme Outcomes:

1. Students will develop a strong foundation in Mathematics capable of competing with their contemporaries in the country.
2. Students will attain the ability to identify, analyse and propose solutions to problems in Mathematics.
3. Students will be motivated and prepared to pursue higher studies and research in Mathematics.
4. Students will be encouraged to dispense their professional and ethical responsibilities.
5. Students will develop confidence for self-education and ability for lifelong learning.
6. Students will get adequately equipped to perform well in competitive exams such as UGC-CSIR NET and GATE.

Programme Specific Outcomes:

1. Students will learn and understand the generalization of the concepts of analysis on $\mathbb{R}$ to general metric spaces and further on to topological spaces.
2. Students will grow familiarised with higher branches of algebra such as ring theory, commutative algebra and Galois theory.
3. Students would be able to learn and appreciate the correlation and unification of varied branches of mathematics at higher levels of the subject.
4. Students would be able to execute numerical computations using computer programming languages.

Course Outcomes

Semester	Course Code	Course Title	Course Outcomes
I	MATC 101	Analysis on Metric Spaces	Students achieve a good command over the concepts in analysis which further serves as a foundation for other courses like complex analysis, topology, calculus and functional analysis.
	MATC 102	Linear Algebra	Students achieve a strong foundation of the concepts in linear algebra which is a prerequisite for further courses like functional analysis and several variable calculus.
	MATC 103	Ordinary Differential Equations	Students understand the different methods of solutions to linear and non-linear differential equations and different approaches used to arrive at a solution.
	MATC 104	Classical Mechanics	Students learn to formulate equations governing different physical systems and solve them using mathematical techniques.
	MATC 105	Analysis on $\mathbb{R}$	Students will develop a strong foundation of the basic topics from undergraduate studies which are indispensable for study of other papers.
II	MATC 201	Algebra	Students understand the fundamental principles and theories concerning basic algebraic structures such as groups, rings, integral domains, fields.
	MATC 202	Complex Function Theory	Students learn to carry over the concepts of real variables to complex variables and learn further results and theories in complex analysis.
	MATC 203	Partial Differential Equations	Students learn different analytical methods to solve different partial differential equations.
	MATC 204	Topology	Students develop the knowledge to understand the different concepts of analysis in arbitrary topological spaces.
	MATC 205	Several Variable Calculus	Students develop the knowledge to extend the concepts of one variable calculus to higher dimensional Euclidean Spaces.
III	MATC 301	Programming in C and	Students develop the ability to formulate efficient algorithms for different mathematical problems and

		Applications	run them in C compiler.
	MATC 302	Computer Practical	Students learn to run the C programs and tackle the errors.
	MATC 303	Measure Theory	Students acquire a general overview of the basic concepts in measure theory and Lebesgue integration.
	MATC 304	Elementary Number Theory	Students acquire a good knowledge of the basic concepts and results in elementary number theory which is essential for higher studies in number theory.
	MATC 305	Theory of Field Extensions	Students learn to extend the algebraic structure of fields and solve algebraic equations using algebraic properties only.
IV	MATC 401	Numerical Analysis	Students understand the analytical approach to the different numerical techniques and apply them to solve different problems.
	MATC 402	Computer Oriented Num Analysis Practical	Students develop C programs for the various numerical methods studied in MATC 401.
	MATC 403	Functional Analysis	Students learn to extend the basic concepts of linear algebra to normed linear spaces and the theories associated with it.
	MATC 404	Project Work	Students will understand their area of work more in depth and be equipped to undertake research work in the area of interest.
	MATC 405	CBCP	-
	Courses for Choice Based Credit Paper (CBCP)		
	MATCBCP 401	Ring Theory	Students learn advanced algebraic structures and their correlation to one another.
	MATCBCP 402	Algebraic Topology	Students acquire knowledge about basic concepts in algebraic topology such as homotopy, fundamental groups and homology.
	MATCBCP 403	Mathematical Methods	Students acquire knowledge about the different mathematical tools and methods which finds application in other fields like engineering and mathematical physics.
	MATCBCP 404	Commutative Algebra	Students acquire knowledge about commutative algebraic structures and the theories associated with

			them.
	MATCBCP 405	Stone-Cech Compactification of a Discrete Semigroup	Students acquire knowledge about structures where both algebraic and topological structures are present.

No. of courses having employability/entrepreneurship/skill development

Course Code	Course Title	Activities having direct bearing on employability/entrepreneurship/skill development	Year of Introduction
MATC 101	Analysis on Metric Spaces	Upon completing these courses, students shall be adequately capable to contribute in mathematics education at all levels of academics, viz, schools, colleges, universities, technical institutes, etc.	2013 and still continuing
MATC 105	Analysis on $\mathbb{R}$		
MATC 201	Algebra		
MATC 301	Programming in C and Applications	Students with a sound knowledge of numerics and number theory enhanced with the capability to work with computer languages can find satisfactory placement in MNCs, IT sectors and government agencies such as ISRO, DRDO, etc.	2013 and still continuing
MATC 304	Elementary Number Theory		
MATC 402	Computer Oriented Numerical Analysis (Practical)		

DETAILS OF SYLLABI

MATC101: Analysis on Metric Spaces

[T = Contact hours (Theory) = 48, P = Contact hours (Practical) = Nil, CR = Credits = 5]

Unit – I

Review of sets, relation and function. Countable and uncountable sets. Metric space: definition and examples. Open balls and closed balls. Interior point, open set, interior of sets, limit point, closed set and closure of sets in metric space. Sequences in metric space: bounded, convergent and Cauchy sequence. Complete metric space.

UNIT – II

Compact metric space. Definition and examples. Theorems on compact spaces. Compact, sequentially compact, limit point compact and their equivalence. Compact sets in Euclidean spaces. Heine – Borel Theorem. Connected metric space. Definition and examples. Theorems on connected spaces. Path connected metric space. Definition and examples. Relationship between connectedness and path connectedness.

Unit – III

Continuous functions on metric spaces. Equivalent characterizations of continuity. Continuous image of compact sets and connected sets. Inverse of continuous functions defined on compact domain. Generalization of the definition of continuity by inverse image of open sets and closed sets. Uniform continuity. Uniformly continuous image of bounded sets and Cauchy sequences.

Unit – IV

Normed linear space. Banach space. Definition and examples. Holder's inequality and Minkowski inequality. Sequences and series of functions: Pointwise convergence and uniform convergence. Cauchy criterion for convergence. Weierstrass M-Test. Uniform convergence and its relationship with continuity, integrability and differentiability. Equicontinuity.

Textbooks:

1. Principles of mathematical analysis, W. Rudin, McGraw Hill Education, 2013.
2. Introduction to topology and modern analysis, G.F. Simmons, 2016
3. Topology of metric spaces, S. Kumaresan, McGraw Hill Education, Narosa, 2005.
4. Mathematical Analysis, T.M. Apostol, Narosa, 2002.
5. Understanding analysis, S. Abbott, Springer, 2015.

COURSE OUTCOME: *Students achieve a good command over the concepts in analysis which further serves as a foundation for other courses like complex analysis, topology, calculus and functional analysis.*

MATC102: Linear Algebra

[T = Contact hours (Theory) = 48, P = Contact hours (Practical) = Nil, CR = Credits = 5]

Unit - I

Vector spaces, linear independence; linear transformations, matrix representation of a linear transformation; isomorphism between the algebra of linear transformations and that of matrices.

Unit - II

Similarity of matrices and linear transformations; trace of matrices and linear transformations, characteristic roots and characteristic vectors, characteristic polynomials, relation between characteristic polynomial and minimal polynomial; Cayley-Hamilton theorem (statement and illustrations only); diagonalizability, necessary and sufficient condition for diagonalizability.

Unit - III

Projections and their relation with direct sum decomposition of vector spaces; invariant subspaces; primary decomposition theorem, cyclic subspaces; companion matrices; a proof of Cayley-Hamilton theorem; triangulability; canonical forms of nilpotent transformations; Jordan canonical forms; rational canonical forms.

Unit - IV

Inner product spaces, properties of inner products and norms, Cauchy-Schwarz inequality; orthogonality and orthogonal complements, orthonormal basis, Gram-Schmidt process; adjoint of a linear transformation; Hermitian, unitary and normal transformations and their diagonalizations.

Forms on inner product spaces and their matrix representations; bilinear forms; Hermitian forms; symmetric bilinear forms; orthogonal diagonalization of real quadratic forms.

Textbooks:

1. Linear Algebra (2nd edition) – K. Hoffman and R. Kunze, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
2. Topics in Algebra (4th edition) – I. N. Herstein, Wiley Eastern Limited, New Delhi, 2013.
3. Linear Algebra, A Geometric Approach – S. Kumaresan, Prentice-Hall of India Pvt. Ltd., New Delhi, 2001.
4. First Course in Linear Algebra – P. B. Bhattacharya, S. K. Jain and S. R. Nagpal, Wiley Eastern Ltd., New Delhi, 2000.
5. Finite Dimensional Vector Spaces – P. R. Halmos, Van Nostrand Inc., 1965.

COURSE OUTCOME: *Students achieve a strong foundation of the concepts in linear algebra which is a prerequisite for further courses like functional analysis and several variable calculus.*

MATC103: Ordinary Differential Equations

[T = Contact hours (Theory) = 48, P = Contact hours (Practical) = Nil, CR = Credits = 4]

UNIT - I

Second Order Linear Equations - The General Solution of the homogeneous equations, the use of a known solution to find another, the homogeneous equation with constant coefficients, the method of undermined coefficients, variation of parameters.

UNIT - II

The oscillations and the Sturm Separation Theorem, the Sturm Comparison theorem, Sturm lioville Boundary problem, Green's function.

UNIT - III

System of First Order Equations - linear systems, homogenous linear system with constant coefficients. Non-linear Equations - Autonomous Systems : The Phase Plane, Types of Critical points and Stability for linear system, Stability by Liapunov's Direct Method, simple Critical Points of Non-linear systems.

UNIT - IV

Existence and uniqueness of solutions : The Method of successive approximations, Picard's theorem, Lipchitz's condition, systems of second order linear Equations .

Textbooks:

1. Differential Equations with Applications and Historical Notes - George F. Simmons, Tata McGraw Hill.
2. Differential Equations - George F. Simmons, Steven G. Krantz, Tata McGraw Hill.
3. An Introduction to Ordinary Differential Equations – E. A. Coddington, Prentice-Hall of India Private Ltd., New Delhi, 2001.
4. Elementary Differential Equations (3rd Edition) – W. T. Martain and E. Relssner, Addison Wesley Publishing Company, inc., 1995.
5. Theory of Ordinary Differential Equations – E. A. Coddington and N. Levinson, Tata McGraw hill Publishing co. Ltd. New Delhi, 1999.

COURSE OUTCOME: *Students understand the different methods of solutions to linear and non-linear differential equations and different approaches used to arrive at a solution.*

MATC104: Partial Differential Equations

[**T** = Contact hours (Theory) = **24**, **P** = Contact hours (Practical) = **Nil**, **CR** = Credits = **2**]

Unit - I

Definition of PDE, origin of first-order PDE; Cauchy's problem for first-order equation, determination of integral surfaces of linear first order partial differential equations passing through a given curve; non-linear PDE of first order, Cauchy's method of characteristic; compatible system of first order PDE; Charpit's method of solution, solutions satisfying given conditions, Jacobi's method of solution.

Unit - II

Origin of second order PDE, linear second order PDE with constant coefficients, separation of variables in a PDE with special reference to Laplace's equation, heat equation and wave equation.

Textbooks:

1. Elements of Partial Differential Equation (3rd edition) – I. N. Sneddon, McGraw Hill Book Company, 1998.
2. Partial Differential Equations (2nd edition) – E. T. Copson, Cambridge University Press, 1995.

COURSE OUTCOME: Students learn different analytical methods to solve different partial differential equations.

MATC105: Analysis on R

[T = Contact hours (Theory) = 48, P = Contact hours (Practical) = Nil, CR = Credits = 4]

Unit – I

The sets **N**, **Z** and **Q**. Algebraic and order properties of **R**. Supremum and infimum. Supremum property and infimum property. Archimedean property and its equivalence with unboundedness of **N**. Interior point, open sets and interior of sets. Limit point, closed sets and closure of sets. Properties of interior and closure of sets. Dense sets. Density of **Q** in **R**.

Unit – II

Sequence: Convergent sequence and limit of convergent sequence. Sandwich lemma. Subsequence, monotone sequence, Cauchy sequence and their properties. Cauchy criterion. Limit superior and limit inferior. Incompleteness of **Q**. Construction of **R**. Infinite series: Partial sums and sum of infinite series. Convergent series. Cauchy criterion. Tests for convergence of series. Absolutely convergent and conditionally convergent series.

Unit – III

Functions. Limit of functions. Uniqueness of limit and theorems on limits. Continuity of functions: $\epsilon - \delta$ definition and sequential definition. Algebra of continuous functions: sum, difference, product, division and composition. Maximum – Minimum theorem. Intermediate value theorem. Uniform continuity. Continuous extension. Monotone function and properties.

Unit – IV

Derivatives. Algebra of differentiable functions: sum, difference, product, division and composition. Rolle's Theorem. Mean Value Theorem. Inverse function theorem. Darboux's theorem for derivatives. Indeterminate forms and L' Hospital's Rule. Maxima and minima. Taylor's Theorem. Riemann integration: Partition and tagged partition. Upper sum, lower sum, upper integral and lower integral. Cauchy criterion for integrability. Properties of the integral. Fundamental theorem of calculus.

Textbooks:

1. Introduction to real analysis, R.G. Bartle and D.R. Sherbert, Wiley, 2011.
2. A basic course in real analysis, A. Kumar and S. Kumaresan, CRC Press, 2014.
3. Analysis – I, T. Tao, Hindustan Book Agency, 2014.
4. Mathematical Analysis, T. M. Apostol, Narosa, 2002.
5. Principles of mathematical analysis, W. Rudin, McGraw Hill Education, 2013.

COURSE OUTCOME: *Students will develop a strong foundation of the basic topics from undergraduate studies which are indispensable for study of other papers.*

MATC201: Algebra

[T = Contact hours (Theory) = 48, P = Contact hours (Practical) = Nil, CR = Credits = 5]

Unit - I

A brief review of groups, their elementary properties and examples, subgroups, cyclic groups, homomorphism of groups and Lagrange's theorem; permutation groups, permutations as products of cycles, even and odd permutations, normal subgroups, quotient groups; isomorphism theorems, correspondence theorem.

Unit - II

Group action; Cayley's theorem, group of symmetries, dihedral groups and their elementary properties; orbit decomposition; counting formula; class equation, consequences for p-groups; Sylow's theorems (proofs using group actions)

Applications of Sylow's theorems, conjugacy classes in S_n and A_n , simplicity of A_n . Direct product; structure theorem for finite abelian groups; invariants of a finite abelian group (Statements only).

Unit - III

Basic properties and examples of ring, domain, division ring and field; direct products of rings; characteristic of a domain; field of fractions of an integral domain; ring homomorphisms (always unitary); ideals; factor rings; prime and maximal ideals, principal ideal domain; Euclidean domain; unique factorization domain.

Unit - IV

A brief review of polynomial rings over a field; reducible and irreducible polynomials, Gauss' theorem for reducibility of $f(x) \in \mathbb{Z}[x]$; Eisenstein's criterion for irreducibility of $f(x) \in \mathbb{Z}[x]$ over $\mathbb{Q}$, roots of polynomials; finite fields of orders 4, 8, 9 and 27 using irreducible polynomials over $\mathbb{Z}_2$ and $\mathbb{Z}_3$.

Textbooks:

1. Topics in Algebra (4th edition) – I. N. Herstein, Wiley Eastern Limited, New Delhi, 2003.
2. A First Course in Abstract Algebra (4th edition) – J. B. Fraleigh, Narosa Publishing House, New Delhi, 2002.
3. Abstract Algebra – D.S. Dummit, R.M. Foote, John Wiley & Sons (2003).
4. Basic Abstract Algebra (3rd edition) – P.B. Bhattacharya, S. K. Jain and S. R. Nagpal, Cambridge University Press, 2000.
5. Contemporary Abstract Algebra (4th edition) – J. A. Gallian, Narosa Publishing House, New Delhi, 1999.

COURSE OUTCOME: *Students understand the fundamental principles and theories concerning basic algebraic structures such as groups, rings, integral domains, fields.*

MATC202: Complex Function Theory

[T = Contact hours (Theory) = 48, P = Contact hours (Practical) = Nil, CR = Credits = 4]

Unit - I

Complex form of equations of straight lines, half planes, circles, etc.; Complex derivatives of complex valued functions of a complex variable, the Cauchy- Riemann equations, holomorphic functions, holomorphic function as mappings; conformal maps; Mobius transformation; cross ratio; symmetry and orientation principle; examples of images of regions under elementary holomorphic functions like exponential function, the complex logarithm.

Unit - II

Brief survey of formal power series, radius of convergence of power series, analytic functions, exponential, cosine and sine, logarithm functions introduced as power series, their elementary properties.

Unit - III

Curves and Contours, Contour Integration, Fundamental theorem of calculus, the Cauchy-Goursat theorem, Cauchy's Integral formula, Liouville's theorem, Maximum Modulus Principle, Zeroes of Analytic/holomorphic functions.

Unit - IV

Taylor's series, Laurent series, Isolated singularities, residues, the Cauchy residue theorem, its application in evaluating real integrals, argument principle and Rouché's theorem.

Textbooks:

1. Complex Function Theory - Donald Sarason, Hindustan Book Agency.
2. Functions of one complex variable – J. B. Conway, Springer International Student edition, Narosa Publishing House, New Delhi, 2000.
3. Elementary theory of Analytic Functions of one or several complex variables- H. Cartan, Courier Dover Publications, New York, 1995.
4. Complex Analysis (2nd Edition) – L. V. Ahlfors, McGraw-Hill International Student Edition, 1990.
5. Complex Variables and Applications – S. Ponnusamy, and H. Silverman, Birkhauser, 2006

COURSE OUTCOME: Students learn to carry over the concepts of real variables to complex variables and learn further results and theories in complex analysis.

MATC203: Fourier Series

T = Contact hours (Theory) = **24**, P =Contact hours (Practical) = **Nil**, CR = Credits =**2**]

Unit-I

Formal power series, real analytic functions, Abel's theorem, multiplication of power series, the substitution theorem, reciprocal of a power series, the exponential and logarithmic functions, trigonometric functions

Unit-II

Periodic functions, inner products on periodic function, trigonometric polynomials, periodic convolution, the Fourier and Plancherel theorems.

Textbooks:

1. Analysis-II (Third Edition), Terrence Tao, HBA.
2. Mathematical Analysis, T. Apostol, Narosa Publishing House.

COURSE OUTCOME: *Students are introduced to the theory of Fourier Series which acts as a prerequisite for higher branches of mathematics such as Fourier Analysis, Functional Analysis and Harmonic Analysis.*

MATC204: Topology

[T = Contact hours (Theory) = **48**, P = Contact hours (Practical) = **Nil**, CR = Credits = **5**]

Unit - I

Definition and examples of topological spaces; basis and sub basis; dictionary order, order topology; subspace topology; Hausdorff spaces, Continuity and related concepts; product topology; quotient topology; the metric topology.

Unit - II

Connected spaces, properties of connected sets; component, path component; local connectedness, local path-connectedness.

Unit - III

Compact spaces; limit point compact and sequentially compact spaces; locally compact spaces; one point compactification; finite product of compact spaces, statement of Tychonoff's theorem, the Stone Cech compactification.

Unit - IV

Countability and separation axioms; Lindelof spaces and separable spaces, Urysohn's lemma; Tietze's extension theorem; statement of Urysohn metrization theorem.

Textbooks:

1. Topology, a first course – J. R. Munkres, Prentice- Hall of India Ltd., New Delhi, 2000.
2. General Topology – J. L. Kelley, Springer Verlag, New York, 1990.
3. An introduction to general topology (2nd edition) – K. D. Joshi, Wiley Eastern Ltd., New Delhi, 2002.
4. Introduction to Topology & Modern Analysis – G. F. Simmons, Tata McGraw Hill.
5. General Topology – J. Dugundji, Universal Book Stall, New Delhi, 1990.

COURSE OUTCOME: *Students develop the knowledge to understand the different concepts of analysis in arbitrary topological spaces.*

MATC205: Several Variable Calculus

[T = Contact hours (Theory) =48, P = Contact hours (Practical) =0, CR = Credits =4]

Unit - I

Functions from $\mathbb{R}^n$ into $\mathbb{R}^m$, Limits and continuity, Derivative, Uniqueness of derivative, Chain rule, Partial derivative, Directional derivative, Continuously differentiable functions, Sufficient condition for a function to be differentiable, Gradient and its relationship with derivative, Partial derivatives of higher order, Schwarz's Theorem.

Unit - II

Mean Value Theorem and Taylor's theorem, Maximum and minimum, Critical points, , Inverse Function Theorem, Implicit Function Theorem, constrained extremes, Lagrange's multipliers.

Unit - III

Sets of measure zero in $\mathbb{R}^n$, Riemann integration in $\mathbb{R}^n$ and its properties, Integration on non-rectangular regions, Multiple integral, Fubini's Theorem, Partitions of unity, Change of variables, Line integral and surface integral.

Unit - IV

Tensor, Tensor product, Wedge product, Differential forms, Poincaré Lemma, Chains, Integration on chains, Manifolds in $\mathbb{R}^n$, Differential forms on manifolds, Tangent space, Orientation, Integration on manifolds, Stokes' theorem on manifolds, Classical Green's Theorem, Divergence Theorem and Stokes' Theorem as consequences of Stokes' theorem on manifolds.

Textbooks:

1. Functions of several variables – Wendell Fleming, Springer-Verlag, New York Inc,, 1977.
2. Calculus on manifolds: A modern approach to classical theorems of advanced calculus – Michael Spivak, CRC Press, 2018.
3. Principles of mathematical analysis – Walter Rudin, McGraw Hill Education, 2013.
4. Analysis on manifolds – James Munkres, CRC Press, 2018.
5. Advanced calculus – GB Folland, Pearson, 2002.

COURSE OUTCOME: *Students develop the knowledge to extend the concepts of one variable calculus to higher dimensional Euclidean Space.*

MATC301: TOPOLOGICAL GROUPS

[T = Contact hours (Theory) = **48**, P = Contact hours (Practical) = **Nil**, CR = Credits =**4**]

Unit-I

Topological group- Definition and examples, homogeneous topological groups, system of neighbourhoods of identity, subgroup, normal subgroup.

Unit-II

Open map, closed map, quotient map, quotient top and quotient space, factor group, homomorphism, isomorphism, direct product of topological groups.

Unit-III

Connected and totally disconnected group and their properties, locally compact group and compact groups and their properties, local isomorphism, topological transformation groups.

Unit-IV

Semi-group, right topological semi-group, compact right topological semi-group, closure and product of ideals, semi-topological and Topological semi-groups.

Textbooks:

1. L.S. Pontryagin Selected Works, Volume II, Topological groups, Gordon and Breach Science Publishers.
2. An Introduction to Topological Groups - P.J. Higgins, Cambridge University Press.
3. A Course on Topological Groups, K. Chandrasekharan, HBA.
4. Topology, James R Munkres, Pearson.

COURSE OUTCOME: *Students develop the knowledge about spaces where both group structures and topological structures are available and how they combine together to give richer structure to the existing ones.*

MATC302: Programming in C (Practical)

[T = Contact hours (Theory) = Nil, P = Contact hours (Practical) = 48, CR = Credits = 2]

The following programs are to be practised:

1. Determination of roots of quadratic equations, $Ax^2+Bx+C=0$,
2. Arranging given set of numbers in increasing/decreasing order, calculation of Mean,
3. Evaluation of sum of power series eg. e^x , $\sin x$, $\cos x$, $\log(1+x)$.
4. Calculation of GCD/LCM of two integers.
5. Evaluation of factorial of a positive integer and evaluation of binomial coefficients.
6. Evaluation of factorial of binomial coefficients mod 2.
7. Sieve method for primality test.
8. Generation of twin primes.
9. Solution of congruence using complete residue system.
10. Evaluation Legendre polynomial from recurrence relation.
11. Addition, subtraction and multiplication of matrices.
12. Transpose, determinant.
13. Inversion of real or complex matrices.
14. Searching a pattern in a given text and replacing every occurrence of it with another given string.
15. Writing a given number in words using function.
16. Arranging a set of names in alphabetical order.
17. Operations with strings and sorting.

COURSE OUTCOME: *Students learn to run the C programs and tackle the errors.*

MATC303: Measure Theory

[**T** = Contact hours (Theory) = **48**, **P** = Contact hours (Practical) = **Nil**, **CR** = Credits = **5**]

Unit - I

σ-algebras, Measures and their properties, outer measures, non-additivity of outer measure, Carathéodory's Theorem Borel measures on the real line, Measurable sets and their properties.

Unit - II

Integration: measurable functions and their characterisations, integration of non-negative functions, the monotone convergence theorem, Fatou's lemma.

Unit - III

Integration of complex functions, the Dominated Convergence theorem, modes of convergence, product measures, the Fubini-Tonelli theorem.

Unit - IV

Signed measures and differentiation - Lebesgue-Radon-Nikodym theorem, functions of Bounded Variation, the Fundamental theorem of Calculus for Lebesgue integrals.

Textbooks:

1. Real Analysis - Modern Techniques and their Application - Gerald B. Folland, John Wiley & Sons Inc.
2. An Introduction to Measure Theory - Terrence Tao, AMS.
3. Measure Theory - P. R. Halmos, Springer-Verlag.
4. Functional Analysis - W. Rudin, Mc-Graw Hill.
5. Real Analysis, third edition, by H. L. Royden, Macmillan Publishing co.inc, New York, 1999.

COURSE OUTCOME: *Students acquire a general overview of the basic concepts in measure theory and Lebesgue integration.*

MATC304: Elementary Number Theory

[T = Contact hours (Theory) = 48, P = Contact hours (Practical) = Nil, CR = Credits = 4]

Unit - I

Divisibility; Euclidean algorithm; primes; congruences; Fermat's theorem, Euler's theorem and Wilson's theorem; Fermat's quotients and their elementary consequences; solutions of congruences; Chinese remainder theorem; Euler's phi-function.

Unit - II

Congruence modulo powers of prime; power residues; primitive roots and their existence; quadratic residues; Legendre symbol, Gauss' lemma about Legendre symbol; quadratic reciprocity law; proofs of various formulations; Jacobi symbol.

Unit - III

Greatest integer function; arithmetic functions, multiplicative arithmetic functions (elementary ones); Mobius inversion formula; convolution of arithmetic functions, group properties of arithmetic functions; recurrence functions; Fibonacci numbers and their elementary properties.

Unit - IV

Diophantine equations-solutions of $ax+by=c$, $x^2+y^2=z^2$, $x^4+y^4=z^2$; properties of Pythagorean triples; sums of two, four and five squares; assorted examples of diophantine equations. Simple continued fractions, finite and infinite continued fractions, uniqueness, representation of rational and irrational numbers as simple continued fractions, rational approximation to irrational numbers, Hurwitz theorem, basic facts of periodic continued fractions and their illustrations (without proofs); Pell's equation.

Textbooks:

1. An Introduction to the Theory of Numbers (6th edition) – I. Niven, H. S. Zuckerman and H. L. Montgomery, John Wiley and sons, Inc., New York, 2003.
2. Elementary Number Theory (4th edition) – D. M. Burton, Universal Book Stall, New Delhi, 2002.
3. History of the Theory of Numbers (Vol. II, Diophantine Analysis) – L. E. Dickson, Chelsea Publishing Company, New York, 1971.
4. An Introduction to the Theory of Numbers (6th edition) – G. H. Hardy and E. M. Wright, The English Language Society and Oxford University Press, 1998.
5. An Introduction to the Theory of Numbers (3rd edition) – I. Niven and H. S. Zuckerman, Wiley Eastern Ltd., New Delhi, 1993.

COURSE OUTCOME: Students acquire a good knowledge of the basic concepts and results in elementary number theory which is essential for higher studies in Number Theory.

MATC305: Theory of Field Extensions

[T = Contact hours (Theory) = 48, P = Contact hours (Practical) = Nil, CR = Credits = 5]

Unit - I

Extension fields, finite extensions; algebraic and transcendental elements, adjunction of algebraic elements, Kronecker theorem, algebraic extensions, splitting fields – existence and uniqueness; extension of base field isomorphism to splitting fields.

Unit - II

Simple and multiple roots of polynomials, criterion for simple roots, separable and inseparable polynomials; perfect fields; separable and inseparable extensions, finite fields; prime fields and their relation to splitting fields; Frobenius endomorphisms; roots of unity and cyclotomic polynomials.

Unit - III

Algebraically closed fields and algebraic closures, primitive element theorem; normal extensions; automorphism groups and fixed fields; Galois pairing; determination of Galois groups, fundamental theorem of Galois theory, abelian and cyclic extensions.

Unit - IV

Normal and subnormal series, composition series, Jordan-Holder theorem (statement only); solvable groups; nilpotent groups.

Solvability by radicals; solvability of algebraic equations; symmetric functions; ruler and compass constructions, fundamental theorem of algebra.

Textbooks:

1. Topics in Algebra (4th edition) – I. N. Herstein, Wiley Eastern Limited, New Delhi, 2003.
2. A First Course in Abstract Algebra (4th edition) – J. B. Fraleigh, Narosa Publishing House, New Delhi, 2002.
3. Contemporary Abstract Algebra (5th edition) – J. A. Gallian.
4. Basic Abstract Algebra (3rd edition) – P. B. Bhattacharya, S. K. Jain and S. R. Nagpal, Cambridge University Press, 2000.
5. Basic Algebra I (3rd edition) – N. Jacobson, Hindustan Publishing corporation, New Delhi, 2002.

COURSE OUTCOME: *Students learn to extend the algebraic structure of fields and solve algebraic equations using algebraic properties only.*

MATC401: Numerical Analysis

[T = Contact hours (Theory) = 48, P = Contact hours (Practical) = Nil, CR = Credits = 4]

Unit - I

A brief introduction to algebraic and transcendental equations and their roots; direct and iterative methods for determination of roots of these equations; initial approximations; bisection method, secant method, Regula-Falsi method, Newton-Raphson method for determination of roots of algebraic and transcendental equations; error analysis, rate of convergence and algorithm for each of these methods.

Unit - II

A brief introduction to systems of linear algebraic equations and their solutions, eigenvalue problem and its solution; direct and iterative methods; forward and backward substitution method; Cramer's rule; Gauss elimination method; Gauss-Jordan elimination method; Gauss-Jacobi iteration method; Gauss-Seidel iteration method; power method for eigenvalue problem; iterative method for matrix inversion; error analysis, rate of convergence and algorithm for each of these methods.

Unit - III

Lagrange and Newton interpolation; Lagrange interpolating polynomial and Newton divided differences interpolating polynomial; linear interpolation; Newton's divided difference interpolation and its generalizations; finite difference operators; relation between differences and derivatives; Gregory-Newton forward and backward difference interpolation; truncation error bounds and algorithm for each of these interpolations.

Unit - IV

Differentiation and integration; numerical differentiation; methods based on linear and quadratic interpolation with error of approximation; methods based on finite differences; optimum choice of step length; numerical integration; methods based on interpolation; determination of the error term; trapezoidal rule; Simpson's rule; error of integration; algorithms for numerical differentiation and integration. ODE and their numerical solutions; IVP ; error estimates; Euler-Richardson method, Runge-Kutta methods and Predictor-Corrector method; error analysis and algorithm for each of these methods; partial differential equations; finite-difference method with error analysis and algorithm.

Textbooks:

1. Numerical Methods for scientific and Engineering computation – M. K. Jain, S. R. K. Iyenger and R. K. Jain, New Age international publishers, New Delhi, 2003.
2. Fundamental of Computer Numerical Analysis – M. Friedman and A. Kandel, CRC Press, Boca Raton, 1993.
3. Applied Numerical Analysis (5th edition) – C. F. Gerald and P. O. Wheatley, Addison-Wesley, New York, 1998.
4. Introduction to Numerical Analysis (2nd edition) – K. E. Atkinson, John Wiley, 1989.
5. Elementary Numerical Analysis: An Algorithmic Approach (3rd edition) – S. D. Conte and C. de Boor, McGraw Hill, New York, 1980.

COURSE OUTCOME: *Students understand the analytical approach to the different numerical-techniques and apply them to solve different problems.*

MATC402: Computer Oriented Numerical Analysis (Practical)

[T = Contact hours (Theory) = Nil, P = Contact hours (Practical) = 48, CR = Credits = 2]

The following programs are to be practised:

1. Solving simple/algebraic/transcendental equations; Newton's method (real roots only),
2. Solutions of system of linear equations, using Gauss' elimination method.
3. Solutions of system of linear equations, using Gauss-Siedel Iterative method.
4. Matrix inversion using Gauss' elimination method.
5. Matrix inversion using Gauss-Jordan method.
6. Power method for finding largest Eigen value.
7. Interpolation using Lagrange's formula.
8. Interpolation using Newton's divided difference formula.
9. Numerical differentiation using Newton's formula.
10. Numerical differentiation using Lagrange's formula .
11. Numerical integration using trapezoidal rule.
12. Numerical integration using Simpson's rules.
13. Improving the numerical integral using Richardson's Extrapolation.
14. Numerical solutions of ordinary differential equations(initial value problems) using Euler-Richardson method.
15. Numerical solutions of ordinary differential equations(initial value problems) using Runge-Kutta method.
16. Numerical solutions of ordinary differential equations(initial value problems) using Predictor-Corrector method.

COURSE OUTCOME: *Students develop C programs for the various numerical methods studied in MATC401.*

MATC403: Functional Analysis

[T = Contact hours (Theory) = 48, P = Contact hours (Practical) = Nil, CR = Credits = 5]

Unit - I

Classical Banach spaces, L^p spaces; Holder's inequality, Minkowski's inequality; convergence and completeness; Riesz-Fischer theorem, bounded linear functional on L^p spaces, Riesz representation theorem.

Unit - II

General Banach spaces – definition and examples; continuous linear transformations between normed linear spaces; Hahn-Banach theorem and its consequences.

Unit - III

Embedding of a normed linear space in its second conjugate space; strong and weak topologies; open mapping theorem; closed graph theorem; uniform boundedness theorem; conjugate of an operator.

Unit - IV

Hilbert's space, examples and simple properties, orthogonal complements, orthonormal set, Bessel's inequalities, complete orthonormal sets, Gram-Schmidt orthogonalization process, self adjoint operators.

Normal and unitary operators, projections, spectrum of an operator, spectral theorem for a normal operator on a finite dimensional Hilbert space.

Textbooks:

1. Real Analysis (4th edition) – H. L. Royden, Macmillan Publishing co. inc, New York, 1999.
2. Introduction to Topology and Modern Analysis (4th edition) – G. F. Simmons, Tata McGraw -Hill Ltd., 2004.
3. Functional Analysis – W. Rudin, Tata McGraw hill Book Company, 1974.
4. Functional Analysis – B. V. Limaye, Wiley Eastern Ltd., 1991.
5. First course in Functional Analysis – C.Goffman and G. Pedrick, Prentice-Hall of India Pvt. Ltd, New Delhi, 1974.

COURSE OUTCOME: *Students learn to extend the basic concepts of linear algebra to normed linear spaces and the theories associated with it.*

MATC404: Project Work

[**T** = Contact hours (Theory) = **48**, **P** = Contact hours (Practical) = **Nil**, **CR** = Credits = **4**]

The project work can be done in any of the areas, namely, Analysis, Algebra, Linear Algebra, Complex Analysis, Functional Analysis, Topology, ODE, PDE, Commutative Algebra, Field Extensions, Number Theory, Several Variable Calculus.

Students will work on problems selected from any of the fields mentioned above.

COURSE OUTCOME: *Students will understand their area of work more in depth and be equipped to undertake research work in this area of interest.*

MATCBP401: Ring Theory

[T = Contact hours (Theory) = **48**, P = Contact hours (Practical) = **Nil**, CR = Credits = **5**]

Unit - I

Basic concepts of rings, modules, operations on ideals and sub-modules; matrix rings, polynomial rings; direct products of rings; fields and division rings; idempotent and nilpotent elements in a ring.

Unit - II

Isomorphism theorems; exact sequences; the group of homomorphisms and its properties relative to exact sequences.

Unit - III

Direct sums and direct products of modules, external and internal direct sums, direct summands; Zorn's lemma, every vector space has a basis; free modules and projective modules; torsion free and torsion modules over commutative domains; exact sequences and projectivity.

Unit - IV

Injective modules, injectivity and divisibility over domains; exact sequences and injectivity; Baer's theorem and its elementary applications; simple modules, semisimple modules (as per Bourbaki); Schur's lemma.

Equivalent conditions for semisimple modules; Wedderburn structure theorem (only statement); characterization of semisimple rings via projective and injective modules.

Textbooks:

1. Elementary Rings and Modules – I. T. Adamson, Oliver and Boyd, Edinburgh, 1995.
2. Notes on Homological Algebra – J. J. Rotman, Van no strand, 1990.
3. Basic Algebra II (3rd edition) – N. Jacobson, Hindustan Publishing Corporation, New Delhi, 2002.
4. Algebra, Second Edition – S. Lang, Addison-Wesley, Massachusetts, 1984.
5. Algebra, Vol. 2: Rings – I. S. Luthar and I.B.S. Passi, Narosa Publishing House, New Delhi, 1999.

COURSE OUTCOME: *Students learn advanced algebraic structures and their correlation to one another.*

MATCBCP402: Algebraic Topology

[T = Contact hours (Theory) = 48, P = Contact hours (Practical) = Nil, CR = Credits = 5]

Unit - I

Homotopy of paths, fundamental group of a topological space, fundamental group functor, homotopy of maps of topological spaces; homotopy equivalence; contractible and simply connected spaces; fundamental group of S^1 , $S^1 \times S^1$ etc.

Unit - II

Calculation of fundamental groups of S^n ($n > 1$) using Van Kampen's theorem (special case); fundamental group of a topological group; Brouwer's fixed point theorem; fundamental theorem of algebra; vector fields, Frobenius theorem on eigenvalues of 3×3 matrices.

Unit - III

Covering spaces, unique lifting theorem, path-lifting theorem, covering homotopy theorem, applications; covering transformations, criterion of lifting of maps in terms of fundamental groups; universal coverings and its existence (statement only).

Unit - IV

Singular homology, reduced homology, Eilenberg-Steenrod axioms (without proof), relation between H_1 and H_2 ; relative homology. Calculus of homology of S^n ; Brouwer fixed point theorem for $f: E_n \rightarrow E_n$ ($n > 2$) and its applications to spheres and vector fields; Meyer-Vietoris sequence and its application.

Textbooks:

1. Topology, a first course – J. R. Munkres, Prentice-Hall of India Ltd., New Delhi, 2000.
2. Algebraic topology, a first course (2nd edition) – M. J. Greenberg and J. R. Harper, Addison-Wesley Publishing co., 1997.
3. Algebraic Topology – A. Hatcher, Cambridge University Press, 2002.
4. Algebraic Topology (2nd edition) – E. H. Spanier, Springer-Verlag, New York, 2000.
5. An Introduction to Algebraic Topology – J. J. Rotman, Graduate Text in Mathematics, No. 119, Springer, New York, 2004.

COURSE OUTCOME: *Students acquire knowledge about basic concepts in algebraic topology such as homotopy, fundamental groups and homology.*

MATCBP403: Mathematical Methods

[T = Contact hours (Theory) = 48, P = Contact hours (Practical) = Nil, CR = Credits = 5]

Unit - I

Laplace transforms, properties of Laplace transform, inversion formula convolution, application to ordinary and partial differential equations; Fourier transform, properties of Fourier transform, inversion formula, convolution, Parseval's equality; Fourier transform of generalized functions, application of transforms to heat wave and Laplace equation.

Unit - II

Formulation of integral equations, integral equations of Fredholm and Volterra type, solution by successive substitution and successive approximation; integral equations with degenerate kernels.

Unit - III

Integral equations of convolution type and their solutions by Laplace transform, Fredholm's theorems; integral equations with symmetric kernel; eigenvalues and eigenfunctions of integral equations and their simple properties.

Unit - IV

Generalized functions; Minusinski's operational calculus of one variable (algebra of addition and convolution of functions, ordered pairs of functions, convolution quotients of a function with a nonzero function), Dirac delta function.

Eigenvalue problem; ordinary differential equations of the Sturm-Liouville type; eigenvalues and eigenfunctions; expansion theorem; extrema properties of the eigenvalues of linear differential operators, formulation of the eigenvalue problem of a differential operator as a problem of integral equation.

Textbooks:

1. Laplace Transform Theory – M. G. Smith, Van Nostrand Inc., 2000.
2. Generalized Functions and Partial Differential Equations – G. E. Shilov, Bernard Seckler, Gordon and Breach, 1999.
3. Integral Equations – David Porter and David S. G. Stirling, Cambridge University Press, 1993.
4. The Use of Integral Transforms – I. N. Sneddon, Tata McGraw Hill, New Delhi, 1974.
5. Lectures on integral equations – H. Widom, Van Nostrand, 1969.

COURSE OUTCOME: *Students acquire knowledge about the different mathematical tools and methods which finds application in other fields like engineering and mathematical physics.*

MATCBP404: Commutative Algebra

[T = Contact hours (Theory) = 48, P = Contact hours (Practical) = Nil, CR = Credits = 5]

Unit - I

Preliminaries on rings and ideals; local and semilocal rings; nilradical and Jacobson radical; operations on ideals; extension and contraction ideals; modules and module homomorphisms; submodules and quotient modules; operations on submodules; annihilator of a module; generators for a module, finitely generated modules; Nakayama's lemma; exact sequences.

Unit - II

Existence and uniqueness of tensor product of two modules; tensor product of n modules; restriction and extension of scalars; exactness properties of tensor products; flat modules.

Unit - III

Multiplicatively closed subsets; saturated subsets; ring of fractions of a ring; localization of a ring; module of fractions and its properties; extended and contracted ideals in a ring of fractions; total ring of fractions of a ring.

Unit - IV

Primary ideals; p -primary ideals; primary decomposition, minimal primary decomposition, uniqueness theorems; primary submodules of a module.

Chain conditions, ascending chain conditions on modules; maximal condition; Noetherian modules; descending chain condition; minimal condition; Artinian modules, their properties; Noetherian rings; Hilbert basis theorem; Artinian rings; structure theorem for Artinian rings.

Textbooks:

1. Introduction to Commutative Algebra – M. F. Atiyah and I. G. Macdonald, AddisonWesley, 2000.
2. Commutative Algebra – N. S. Gopalakrishnan.
3. Undergraduate Commutative Algebra – M. Reid, London Math. Soc. Student Texts, No. 29, 1995.
4. Algebra (Volume 2: Rings) – I. S. Luther and I. B. S. Passi, Narosa Publishing House, New Delhi, 1999.
5. Algebra (Volume 3: Modules) – I. S. Luther and I. B. S. Passi, Narosa Publishing House, New Delhi, 1999.

COURSE OUTCOME: *Students acquire knowledge about commutative algebraic structures and the theories associated with them.*

MATCBCP405: Stone-Cech Compactification of a Discrete Semigroup

[T = Contact hours (Theory) = 48, P = Contact hours (Practical) = Nil, CR = Credits = 5]

Unit - I

Semigroup, idempotents, subgroups, ideals, idempotents and order, minimal left ideals and idempotents, Right Topological semigroup, compact right topological semigroup and its properties, closure and product of ideals.

Unit - II

The Stone-Čech compactification, Ultrafilters and their characterizations, principal ultrafilters and non-principal ultrafilters and their properties, the topological space βD and the basis for the topology on βD , Stone-Cech Compactification of a Discrete topological Space and its properties, Uniform limits via ultrafilters.

Unit - III

Extending the operation of a semigroup S to βS , definition and characterization of the members of $p \cdot q$ where p, q are elements of βS , commutativity in βS , algebra of non-principal ultrafilters.

Unit - IV

Syndetic sets, piecewise syndetic sets, thick sets and their characterizations, relation with $K(\beta S)$, central Sets-definition and characterizations, interrelationship among the different notions of largeness of a set, Hindman's theorem.

Textbooks:

1. Algebra in Stone-Cech Compactification, Neil Hindman, Dona Strauss, 2012, De Gruyter.
2. Topics in Algebra of βS , P Adams, PhD Dissertation(2001), Hull University.
3. The Stone-Cech Compactification of a topological semigroup, J. Baker and R. Butcher, Math Proc. Cambridge Philosophical Society 80 (1976),103-107
4. Analysis on semigroups, J Berglund, H. Junghenn and P. Milnes, Wiley, N. Y. 1989
5. Recurrence in ergodic theory and combinatorial number theory, H. Furstenberg, Princeton University Press, Princeton, 1981

COURSE OUTCOME: *Students acquire knowledge about structures where both algebraic and topological structures are present.*